The Oaxaca-Blinder decomposition in Stata: an update

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Outline

Introduction

- In 2008, I published Stata command `oaxaca`, which implements the Oaxaca-Blinder (OB) decomposition technique (Jann 2008).
- The OB decomposition (Blinder 1973, Oaxaca 1973) is used to analyze differences in outcomes between groups, such as the wage gap by gender or race (for a general overview of counterfactual decomposition methods see Fortin et al. 2011).
- The technique is highly popular in applied research (over 10'000 citations of both Oaxaca 1973 and Blinder 1973 on Google Scholar; about 3000 citations of Jann 2008).
- Over the years, both the functionality of Stata and the literature on decomposition methods have evolved, so that an update of the `oaxaca` command is long overdue.

Desired features

- 👍 Overall and detailed decompositions supporting different solutions to the index problem (see, e.g., Jann 2008).
- 👍 Variance estimation (Jann 2008).
  - Support for survey estimation (`pweights`, clustered standard errors, general support for `svy`).
  - Provided by existing `oaxaca`, but there is scope for improvement.
- 👍 Support for binary dependent variables (Yun 2004)
- 👍 „Normalization“ for categorical predictors (Yun 2008)

(👍 = supported by current version of `oaxaca`; 👎 = currently not supported)

Desired features

- Support for factor variables.
- Support for more than two groups (series of decompositions against a reference group or an overall average).
- Alternative "normalization" approaches (Kim 2013, Horrace and Oaxaca 2001).
- Decompositions based on reweighted techniques (DiNardo et al. 1996) such as IPW or entropy balancing (Hainmueller 2012).
- Decompositions for arbitrary statistics (rather than just the mean) based on recentered influence functions (RIF) (Firpo et al 2009, 2018, Rios-Avila 2020).
- Support for difference-in-differences decompositions (Smith and Welch 1987, Kröger and Hartmann 2021).

Desired features

- There are further decomposition approaches for which an integration into `oaxaca` appears be less obvious. For example:
  - Fairlie (2005) decomposition for binary dependent variables (see Jann 2006 for an implementation).
  - Juhn et al. (1991, 1993) decompositions based on residual distributions (see Jann 2005a and 2005b for implementations).
  - Distributions based on quantile regression process or distribution regression (Chernozhukov et al. 2013; see Jann 2023 for an implementation).

Methods

- The general idea of counterfactual decomposition methods is to decompose a group difference in a distributional statistic (Δ^ν) into a part that is related to compositional differences between the groups (Δ^ν_X) and a part that is related to group-specific "mechanisms" (structural functions) (Δ^ν_S).

$$\Delta^\nu = \Delta^\nu_X + \Delta^\nu_S$$

- The classical Oaxaca-Blinder decomposition (a) focuses on the mean and (b) uses linear regression for the structural function. In its simplest form, it can be written as

$$\underbrace{\bar{Y}^1 - \bar{Y}^2}_{\widehat{\Delta}^\mu} = \underbrace{(\bar{X}^1 - \bar{X}^2)\widehat{\beta}^1}_{\widehat{\Delta}^\mu_X} + \underbrace{\bar{X}^2(\widehat{\beta}^1 - \widehat{\beta}^2)}_{\widehat{\Delta}^\mu_S}$$

where $\bar{Y}^g$ is the mean of the outcome, $\bar{X}^g$ is the mean vector of characteristics, and $\widehat{\beta}^g$ is the coefficient vector of a regression of Y on X in group g.

Methods

- Variants of the classical decomposition differ in how exactly the group means and coefficients are combined to form the two terms (and some variants also have a third term), but the basic principle stays the same.
- In case of reweighting, weights are computed that balance the distribution of characteristics between groups, and a (four-term) decomposition is obtained by comparing weighted and unweighted results.
- In case of RIF decomposition, Y is replaced by the (group-specific) recentered influence function of statistic $\nu(F_Y)$ (e.g. the RIF of the Gini coefficient of Y). All else stays the same.
- In case of a difference-in-differences decomposition, an additional group layer (e.g. two time points) is added and additional terms are defined, but the logic stays the same.

Methods

- The basic message is that we can put all of the above into a common framework without much conceptual complication.
- Variance estimation (taking account of reweighting and including support for `svy`) can easily be implemented using influence functions (see Jann 2019, 2020b, 2021).
- The basic elements we need are:
  - Mean estimates (influence function = demeaned variable).
  - Coefficients from regression models (influence functions for linear regression and maximum likelihood estimators are very easy to obtain; just need the scores and the information matrix).
  - Recentered influence function for the statistic of interest (a wide variety of RIFs is provided by command `dstat` by Jann 2020a).
- However, as usual, there are many little details to take care of.

Syntax

New `kob` command:[1]

`kob` *statistic depvar* [*indepvars*] [*if*] [*in*] [*weight*] ,
`by(`*groupvar* [*groupvar2*]`)`
[`reweight`[`(`*varlist*`)`] `vce(`*vcetype*`)` *options*]

- *statistic*: any statistic allowed by `dstat`
- *groupvar2*: for DID decomposition
- `reweight()`: apply reweighting
- *vcetype*: `robust`, `cluster`, `svy`, `bootstrap`, `jackknife`
- *options*: type of decomposition, reporting, etc.

[1] `kob` = Kitagawa-Oaxaca-Blinder (see Kitagawa 1955); the name of the command may still change.

Example: Private–public gap in wage inequality

Data from the German Socio-Economic Panel (GSOEP), wave 2015.

```
. use gsoep-extract, clear
(Example data based on the German Socio-Economic Panel)
. keep if wave==2015
(29,970 observations deleted)
. keep if inrange(age, 25, 55)
(5,671 observations deleted)
. generate lnwage = ln(wage)
(1,709 missing values generated)
. summarize public wage lnwage yeduc expft weight psu
```

Variable	Obs	Mean	Std. dev.	Min	Max
public	5,770	.2353553	.4242574	0	1
wage	5,600	17.57278	9.858855	3.03	121.42
lnwage	5,600	2.736721	.5062968	1.108563	4.799255
yeduc	7,121	12.28823	2.783974	7	18
expft	7,274	11.63359	9.556508	0	39.5
weight	7,309	2204.229	3025.122	3.3	32681.6
psu	7,309	2437.243	1413.001	1	4893

Private–public wage gap

Current oaxaca implementation:

```
. generate expft2 = expft^2
(35 missing values generated)
. oaxaca lnwage yeduc expft expft2 [pw=weight], by(public) weight(1) ///
>     nodetail vce(cluster psu)
Blinder-Oaxaca decomposition                    Number of obs     =      5,458
                                                Model             =     linear
Group 1: public = 0                             N of obs 1        =      4,184
Group 2: public = 1                             N of obs 2        =      1,274
    explained: (X1 - X2) * b1
  unexplained: X2 * (b1 - b2)
                                (Std. err. adjusted for 2,036 clusters in psu)
```

lnwage	Coefficient	Robust std. err.	z	P>|z|	[95% conf.	interval]
overall						
group_1	2.732109	.0139572	195.75	0.000	2.704754	2.759465
group_2	2.866068	.0213964	133.95	0.000	2.824132	2.908005
difference	-.1339592	.0249932	-5.36	0.000	-.182945	-.0849735
explained	-.1262644	.0170697	-7.40	0.000	-.1597204	-.0928084
unexplained	-.0076948	.0226291	-0.34	0.734	-.0520471	.0366575

Private–public wage gap

New `kob` command:

```
. kob mean lnwage yeduc c.expft##c.expft [pw=weight], by(public) vce(cluster psu)
Kitagawa-Oaxaca-Blinder decomposition           Number of obs     =      5,458
                                                Statistic         =       mean
                                                Model             =     linear
Group 1: public = 0                             N of obs 1        =      4,184
Group 2: public = 1                             N of obs 2        =      1,274
delta_X: (X1 - X2) * b1
delta_S: X2 * (b1 - b2)
                                (Std. err. adjusted for 2,036 clusters in psu)
```

lnwage	Coefficient	Robust std. err.	z	P>|z|	[95% conf.	interval]
levels						
group_1	2.732109	.0141087	193.65	0.000	2.704457	2.759762
group_2	2.866068	.0221403	129.45	0.000	2.822674	2.909463
g1_vs_g2						
gap	-.1339592	.0256495	-5.22	0.000	-.1842314	-.0836871
delta_X	-.1262644	.0171534	-7.36	0.000	-.1598845	-.0926443
delta_S	-.0076948	.0226074	-0.34	0.734	-.0520046	.0366149

Private–public gap in wage inequality

Gini coefficient:

```
. kob gini wage yeduc c.expft##c.expft [pw=weight], by(public) vce(cluster psu)
Kitagawa-Oaxaca-Blinder decomposition           Number of obs     =      5,458
                                                Statistic         =       gini
                                                Model             =     linear
Group 1: public = 0                             N of obs 1        =      4,184
Group 2: public = 1                             N of obs 2        =      1,274
delta_X: (X1 - X2) * b1
delta_S: X2 * (b1 - b2)
                                (Std. err. adjusted for 2,036 clusters in psu)
```

wage	Coefficient	Robust std. err.	z	P>|z|	[95% conf.	interval]
levels						
group_1	.2783233	.0056676	49.11	0.000	.267215	.2894316
group_2	.2213006	.0081333	27.21	0.000	.2053596	.2372415
g1_vs_g2						
gap	.0570227	.0098305	5.80	0.000	.0377553	.0762901
delta_X	-.0093274	.0048026	-1.94	0.052	-.0187404	.0000856
delta_S	.0663501	.0109198	6.08	0.000	.0449477	.0877525

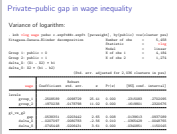
Private–public gap in wage inequality

Variance of logarithm:

```
. kob vlog wage yeduc c.expft##c.expft [pw=weight], by(public) vce(cluster psu)
Kitagawa-Oaxaca-Blinder decomposition           Number of obs     =      5,458
                                                Statistic         =       vlog
                                                Model             =     linear
Group 1: public = 0                             N of obs 1        =      4,184
Group 2: public = 1                             N of obs 2        =      1,274
delta_X: (X1 - X2) * b1
delta_S: X2 * (b1 - b2)
                                (Std. err. adjusted for 2,036 clusters in psu)
```

wage	Coefficient	Robust std. err.	z	P>|z|	[95% conf.	interval]
levels						
group_1	.2508589	.0098729	25.41	0.000	.2315083	.2702095
group_2	.1970238	.0178798	11.02	0.000	.1619801	.2320676
g1_vs_g2						
gap	.0538351	.0203442	2.65	0.008	.0139613	.0937089
delta_X	-.0207097	.0080783	-2.56	0.010	-.0365429	-.0048765
delta_S	.0745448	.0206431	3.61	0.000	.0340851	.1150045

Private–public gap in wage inequality



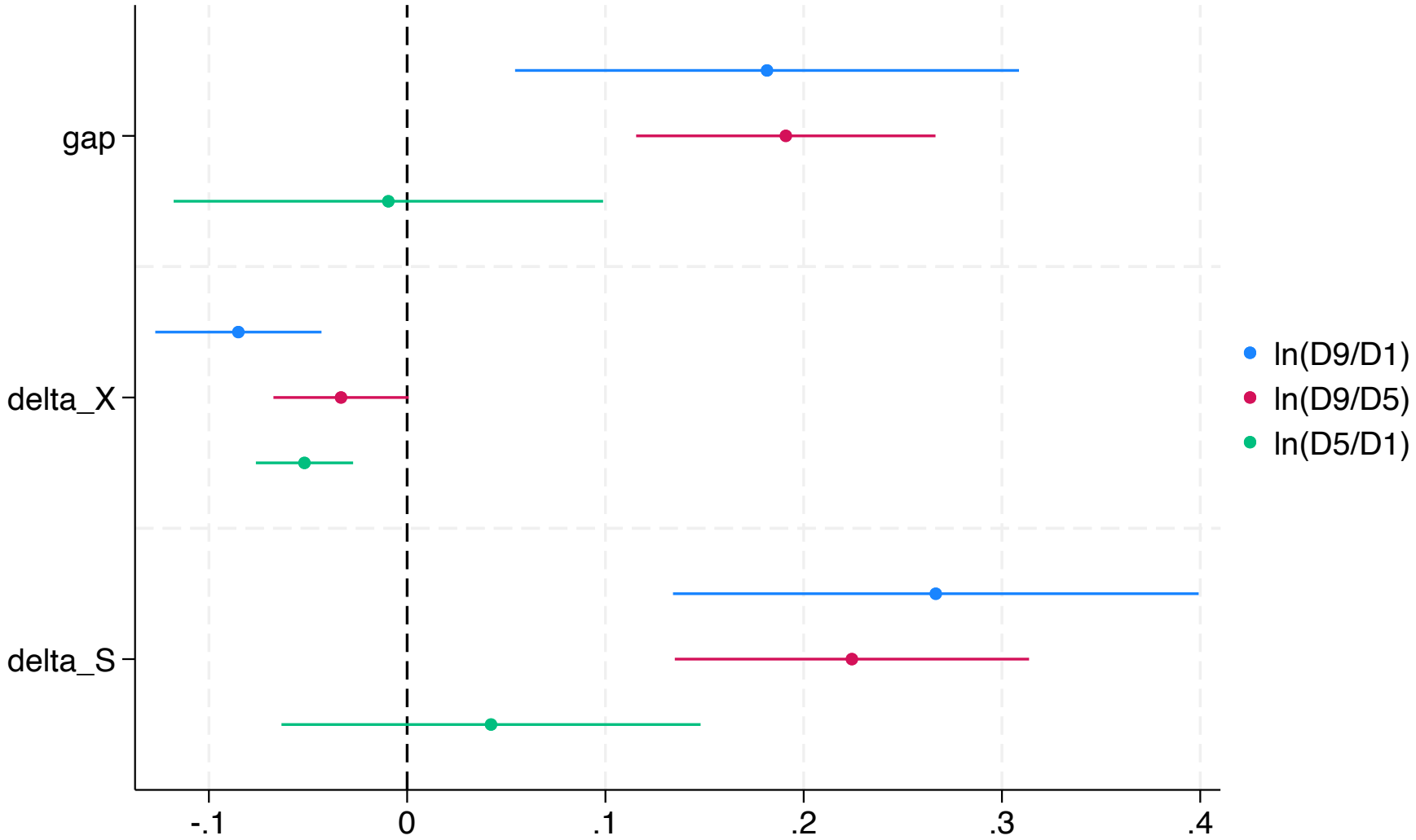
Could also type:

```
. kob variance lnwage yeduc c.expft##c.expft [pw=weight], by(public) vce(cluster psu)
Kitagawa-Oaxaca-Blinder decomposition           Number of obs     =      5,458
                                                Statistic         =   variance
                                                Model             =     linear
Group 1: public = 0                             N of obs 1        =      4,184
Group 2: public = 1                             N of obs 2        =      1,274
delta_X: (X1 - X2) * b1
delta_S: X2 * (b1 - b2)
                                (Std. err. adjusted for 2,036 clusters in psu)
```

lnwage	Coefficient	Robust std. err.	z	P>|z|	[95% conf.	interval]
levels						
group_1	.2508589	.0098729	25.41	0.000	.2315083	.2702095
group_2	.1970238	.0178798	11.02	0.000	.1619801	.2320676
g1_vs_g2						
gap	.0538351	.0203442	2.65	0.008	.0139613	.0937089
delta_X	-.0207097	.0080783	-2.56	0.010	-.0365428	-.0048765
delta_S	.0745448	.0206431	3.61	0.000	.0340851	.1150045

Private–public gap in wage inequality

Quantile ratios:



Private–public gap in wage inequality


```
kob iqr(10,90) lnwage yeduc c.expft##c.expft [pw=weight], by(public) vce(cluster psu)
est sto d9d1
kob iqr(50,90) lnwage yeduc c.expft##c.expft [pw=weight], by(public) vce(cluster psu)
est sto d9d5
kob iqr(10,50) lnwage yeduc c.expft##c.expft [pw=weight], by(public) vce(cluster psu)
est sto d5d1
coefplot d9d1 d9d5 d5d1, keep(g1_vs_g2:) xline(0) plotl(ln(D9/D1) ln(D9/D5) ln(D5/D1))
```

Conclusions

- A general and flexible command for Oaxaca-Blinder decompositions, including RIFs and reweighting as well as support for survey estimation, is straightforward to implement (at least conceptually).
- First steps have been taken . . .
- . . . but I am not quite done yet.
- I was too busy working on `geoplot`.

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